Introduction to Lattice QCD
Lecture 5

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The continuum limit

Very general, very important:
A continuum limit can be defined by tuning the bare parameter to the critical surface of a lattice system. As long as the critical behavior is universal the lattice continuum limit is universal as well. Give up one physical quantity for every relevant parameter - everything else is a prediction.

Smearing vs blocking
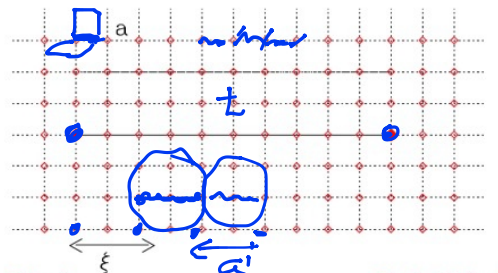
Smearing: $U_{n,\mu} \to W_{n,\mu}(U),\ a = a',\ \xi = \xi'$
Blocking: $U_{n,\mu} \to W_{2n,\mu}(U),\ 2a = a',\ \xi = 2\xi'$
In both procedures the IR (long distance) physics is unchanged

Smearing vs blocking

$$Z = \int \mathcal{D}U e^{-S[U]} = \int \mathcal{D}U \mathcal{D}V \underbrace{\prod_n \delta(V_n - W_{n\mu}(U)) e^{-S[U]}}_{\Uparrow}$$

$$= \int \mathcal{D}V e^{-S'[V]}$$

$S'[V]$ is some complicated (but local!) new action
Example-I: start from nearest-neighbor scalar $\to$ add nun, diagonal,etc.
Example-II: start from plaquette gauge $\to$ plaquette + 6-link +8link + .
Flow in action space

Smearing vs blocking

Smearing

$$a \qquad = a'$$
$$m_{phys} = m_{latt}/a \quad = m'_{latt}/a'$$
$$\xi_{latt} \quad = \xi'_{latt}$$

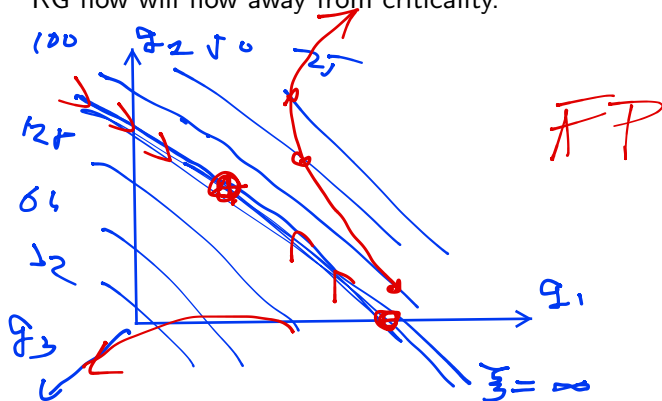$S[V]$ stays on the "constant (lattice) physics" surface

Blocking

$$a \qquad = a'/2$$
$$m_{phys} = m_{latt}/a \quad = m'_{latt}/a'$$
$$\xi_{latt} \quad = 2\xi'_{latt}$$

$S[V]$ flows away from criticality

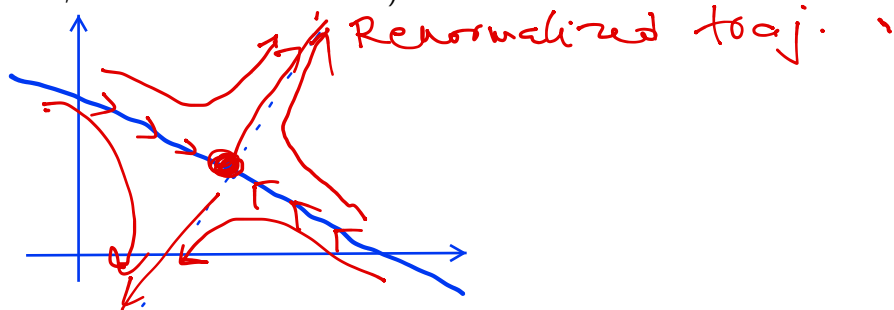$$S[u] \longrightarrow S_1[v_1] \longrightarrow S_2[v_2] \longrightarrow \ldots$$

Fixed points

Blocking can have fixed points $S[V] = S'[V]$ only if $\xi = \infty$ or 0.
RG flow will flow away from criticality.

Fixed points

Assumption : the block transformation has a fixed point. (Most does; some "sick" ones do not.)

Renormalized traj.

Relevant operators: flow away from the critical surface, have to be tuned to criticality

Irrelevant operators: for into the FP, initial value is irrelevant

Wilson renormalization group

Interpretation:

Bare coupling : $g_0 = g(\Lambda_{cut} = \pi/a)$

After RG blocking that changes $a \to a^* e^t$

$g_0 \to g(\mu = \Lambda_{cut} e^{-t})$: the running coupling at energy μ!

How fast does the coupling run?

$$\mu \frac{dg}{d\mu} = \mu \frac{dg}{dt} \left(\frac{d\mu}{dt} \right)^{-1} = -\frac{dg}{dt}$$

A relation between bare parameters and renormalized ones.

Universality

Irrelevant operators are irrelevant: it does not matter what the original action is, we will end up on the renormalized trajectory every time. That is why we can discretize the action, choose the favorite improvement (or lack of): lattice artifacts are just irrelevant operators.

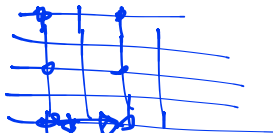Except when one starts too far, in the wrong basin of attraction.

Where is the Fixed Point?
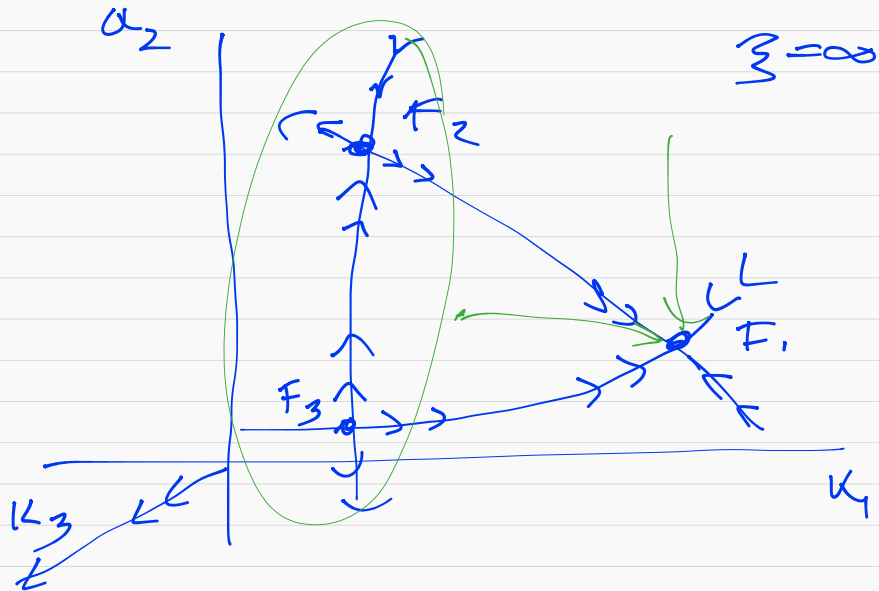
In our imagination..

Really.

Good block transformations have FP's, but each one has a different FP: there is nothing specific about the FP's locations, i.e. no physical observable is going to show it.
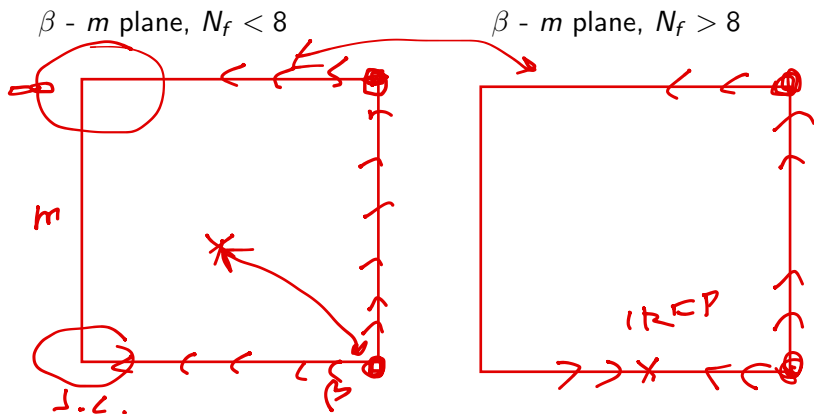
α_2

$\xi = \infty$

F_2

L

F_1

F_3

k_3

u_1

The fixed point structure of QCD

β - m plane, $N_f < 8$

β - m plane, $N_f > 8$

m

J.C.
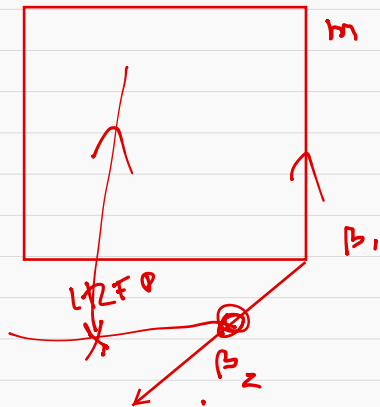
IREP

Warning: $N_f^* = 8.05$ for conformal FP is only a 2-loop perturbative prediction. It could due more or less.

Extend the coupling space

m

B_1

$L \frac{1}{2} F \theta$

B_2
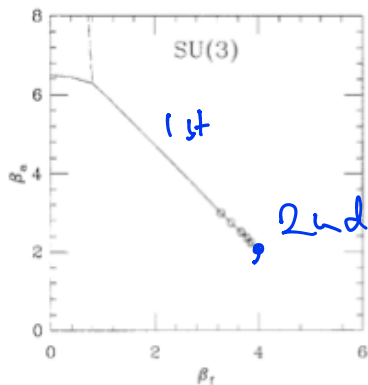
Spurious fixed points

Lattice artifacts can introduce spurious fixed points that change lattice predictions. Avoid them. Example: Pure gauge action in fundamental-adjoint plaquette space:
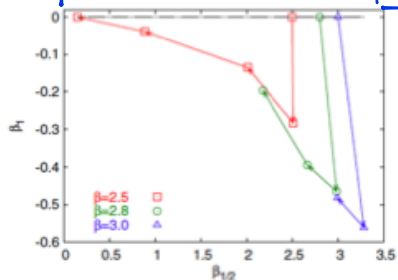
$$S = \frac{\beta_f}{N} \sum_p (N - \mathrm{ReTrU}_\square) + \frac{\beta_a}{N}(N - \mathrm{ReTrU}_\square)^2$$
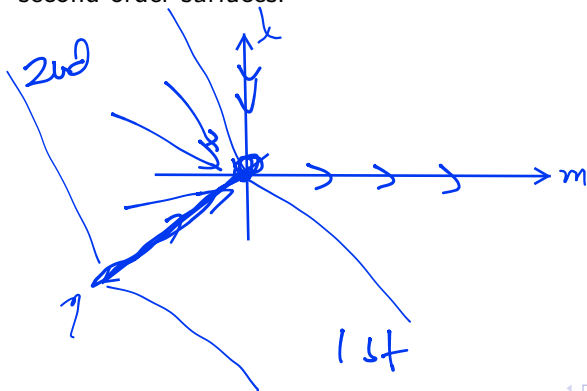
Spurious fixed points

RG flow in pure gauge action in fundamental-adjoint plaquette space (T. Tomboulis et al)

The triviality of the scalar model

$$S[\phi] = \sum_n \frac{1}{2}(\Delta_\mu \phi_n)^2 + \underbrace{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\eta}{6}\phi^6}$$

Fixed point is at $m = \lambda = \eta = 0$. Which operators are relevant? In 4 dimensions only m. Yet the phase structure has 1st order & second order surfaces.

If it is not QCD

QCD is the only part of the Standard Model that is renormalizable on its own: the $g^2 = 0, m = 0$ fixe point is UV repulsive, by tuning the bare couplings interacting renormalized theories are obtained. But the SM cannot stand on its own. (Even if you don't believe in naturalness.)

So what's beyond SM? What drives the electroweak symmetry breaking?

There are theoretical suggestions that gauge-fermion systems can explain the Higgs mechanism, replacing elementary scalar by composite fermions. These models are based on the old technicolor idea, but are far beyond it. And they are all strongly coupled, requiring non-perturbative investigations.

Technicolor

Imagine there is a yet undiscovered sector with 3 massless techni-quarks gauged by SU(3) : this is chirally broken, like QCD. The pions are massless Goldstones - just what the $W^{\pm}$ and Z bosons are waiting for! This could break the electroweak symmetry if f_{Π} 250 GeV, the electroweak scale. (Scaled up QCD)

Scaled-up QCD violates electroweak measurements, but the idea is good. Maybe other gauge & fermion models could do this job. What's required? A walking theory, with β function like
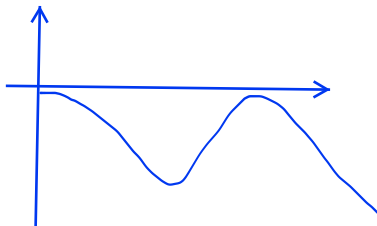
Technicolor

Are there walking theories? Maybe.
Can they predict a 125GeV scalar (is it indeed scalar?)? Yes, there are several options.
What can lattice do?
Study different models: are they conformal? Are they confining? Any of the walking?
Can these models be in agreement with electroweak measurements? (FCNC, S parameter, a 125GeV scalar , etc)?
Plenty of questions, fun calculations.

Enjoy.

ENJOY!

and thank you for
all the questions, comments, and
attention