High Temperature and Density in Lattice QCD: Connection with phenomenology

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Hadron resonance gas model

- Simple (simple-minded) model often used for low T comparison.
- Hagedorn: A noninteracting gas of mesons and baryons including resonances.
- Include all particles (and resonances) listed in the PDG summary.
- Stop at a some cut off mass M.
- Interactions are treated only by including resonances
- Should be good for $T \ll m_\pi$, the lowest mass.
- If the density of states grows as $dN/dm = C \exp(m/T_c)$ then the partition function diverges for $T > T_c$ (limiting temperature)
- Best to switch to quark and gluon degrees of freedom for $T > T_c$.

Hadron resonance gas model

- Low temperature limit
- An explicit expression for the partition function for mesons/baryons (M/B)
- $\log \mathcal{Z} = \sum_i \log \mathcal{Z}^{\mathcal{M}} + \sum_i \log \mathcal{Z}^{\mathcal{B}}$.
- For the ith meson or baryon we have

$$\begin{aligned}\log \mathcal{Z}^{M/B}_{M_i} &= \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \log(1 \mp z_i e^{-\varepsilon_i/T}) \\ &= \frac{VT^3}{2\pi^2} d_i \left(\frac{M_i}{T}\right)^2 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{z_i^k}{k^2} K_2(kM_i/T) \quad .\end{aligned}$$

- d_i is a multiplicity factor.

Ideal gas limit (Stefan-Boltzmann)

- The Stefan-Boltzmann gas is often used for comparison at high temperature.
- Assumes free, massless quarks and gluons.
- Pressure with chemical potential μ_f.

$$\frac{p_{SB}}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{20} + \sum_{f=u,d,s} \left[\frac{1}{2} \left(\frac{\mu_f}{T} \right)^2 + \frac{1}{4\pi^2} \left(\frac{\mu_f}{T} \right)^4 \right] \quad ,$$

Equation of state in general

- Energy density and pressure

$$\varepsilon = \frac{T^2}{V}\frac{\partial \log Z}{\partial T}\bigg|_V$$
$$p = T\frac{\partial}{\partial V}\log Z\bigg|_T$$

- To calculate them separately is more involved. It is more convenient to calculate the interaction measure

$$I = \varepsilon - 3p = -\frac{T}{V}\frac{d\log Z}{d\log a}.$$

- For the Wilson gauge action we get

$$I = -T/V(d\log g^2/d\log a)\langle S_G\rangle.$$

- We must subtract the vacuum value to remove a UV divergence. From now on, we assume this has been done and drop the Δ.

$$\Delta I = I(T) - I(0)$$

Exercise

The previous slide gives the thermodynamic identities that relate the energy density and pressure to derivatives of the ensemble free energy with respect to temperature and volume, respectively. On a lattice of a fixed number of sites $N_s^3 \times N_t$*, the volume is given in terms of the spatial and temporal lattice constants* a_s *and* a_t *by* $N_s^3 a_s^3$*, and the inverse temperature is given by* $a_t N_t$*. So we can relate the derivatives in the thermodynamic identities to derivatives with respect to* a_s *and* a_t*. To relate these derivatives to the lattice action, one must take care to include the appropriate factors of* a_s *and* a_t *in the expression for the lattice action and to remember that the gauge coupling* g^2 *also depends on the lattice constants.*

With these preliminaries in mind, show that

$$I \equiv \varepsilon - 3p = -T/V(d \log g^2/d \log a) \langle S_G \rangle .$$

Equation of state in general

- Thermodynamic identity for the pressure $\log Z = -pV/T$.

$$I = \frac{T}{V}\frac{d(pV/T)}{d\log a},$$

- Integrate from low temperature (large $a = a_0$) to high (small a).

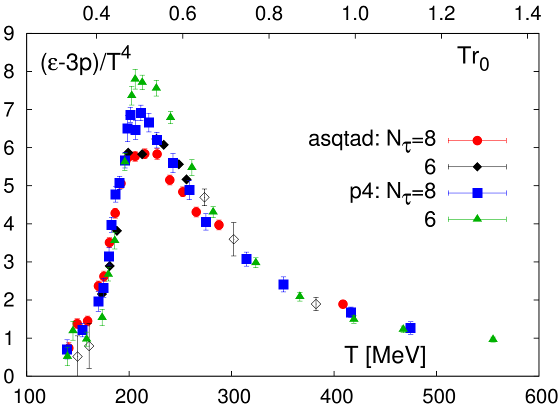
$$p(a)a^4 - p(a_0)a_0^4 = -\int_{\log a_0}^{\log a} \Delta I(a')(a')^4\, d\log a'.$$

- At low enough T_0 we may take $p(a_0) = 0$.

Exercise

Derive the integral expression for the pressure that was given in the previous slide.

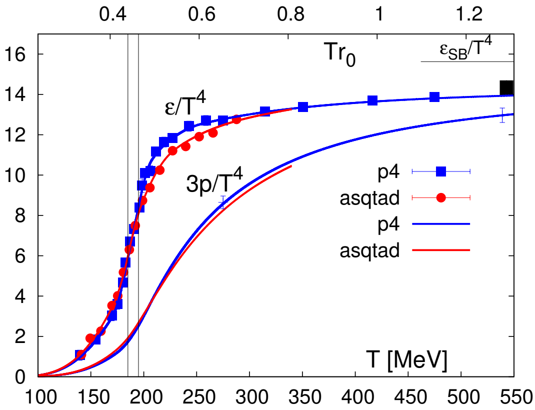
Interaction measure



[HotQCD, Phys. Rev. D80 (2009) 014504]

- As the lattice spacing is decreased the peak softens a bit.

Energy density

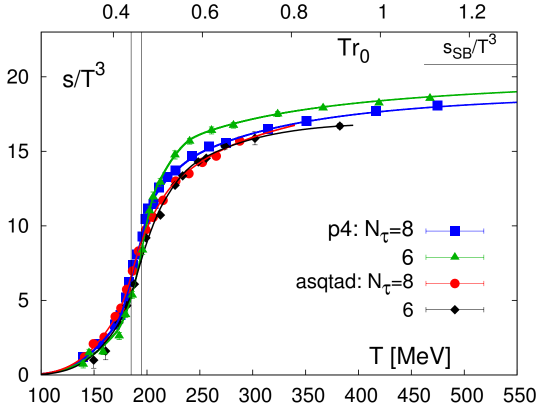


[HotQCD, Phys. Rev. D80 (2009) 014504]

- The vertical bars are at 190-195 MeV. These results are for higher than physical mass and for nonzero lattice spacing.

Entropy density

- Entropy density $s = \varepsilon + p$

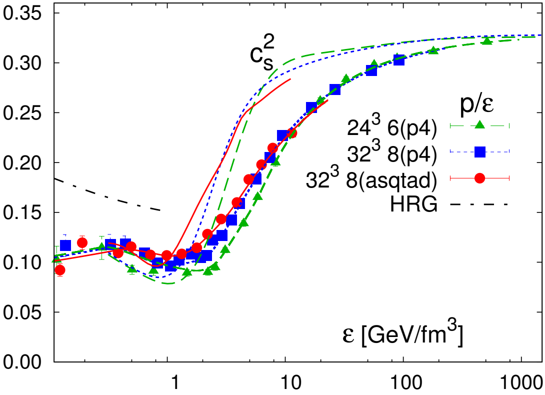


[HotQCD, Phys. Rev. D80 (2009) 014504]

Speed of sound

- Speed of sound

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon\frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon}\ .$$



[HotQCD, Phys. Rev. D80 (2009) 014504]

Nonzero density

- We can't simulate directly at $\mu \neq 0$ because the fermion determinant is complex.
- For heavy ion collisons, the chemical potentials are small.
- Taylor series expansion $(2+1$ flavor case)

$$\frac{p}{T^4} = \sum_{n,m=0}^{\infty} c_{nm}(T) \left(\frac{\mu_{ud}}{T}\right)^n \left(\frac{\mu_s}{T}\right)^m ,$$

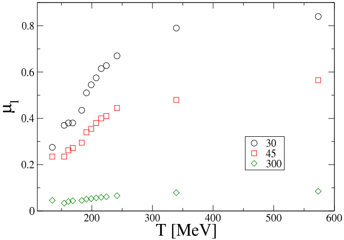
- The coefficients are evaluated at $\mu_{ud} = \mu_s = 0$

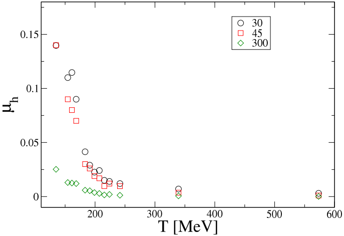
$$c_{nm}(T) = \frac{1}{n!}\frac{1}{m!}\frac{1}{T^3 V}\frac{\partial^{n+m}\log Z}{\partial(\mu_{ud}/T)^n \partial(\mu_s/T)^m}\bigg|_{\mu_{ud,s}=0} .$$

- The derivatives are expectation values of combinations of traces of the inverse of the lattice Dirac matrix.

Nonzero density

- In heavy ion collisons the strange number density n_s is zero.
- Tune $\mu_\ell = \mu_u = \mu_d$ and μ_s to get $n_s = m_B = 0$

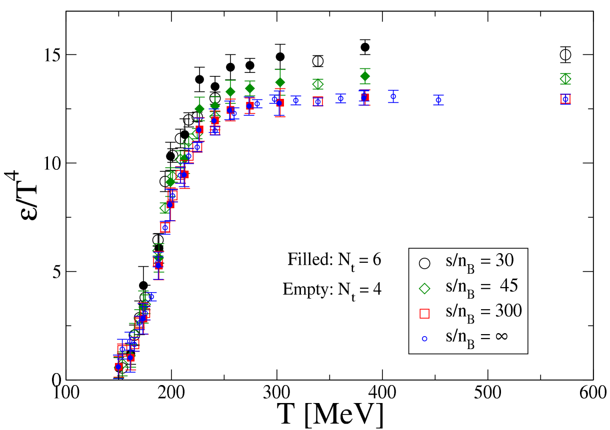




[MILC Phys.Rev. D77 (2008) 014503]

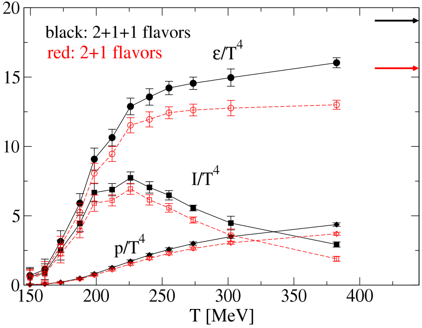
- s/n_B is the ratio of entropy density to baryon density.
- (Often assume isentropic formation, expansion of the plasma.)

Nonzero density



[MILC, Phys.Rev. D81 (2010) 114504]

Charm contribution to EOS



[MILC, 2010]

- Charm might not have time to reach equilibrium in a heavy-ion collision.
- Charm effects start to become visible above about $T = 200$ MeV.
- Stout and p4 action results are a bit smaller than this.

Fluctuations

- In a neutral ensemble, conserved charges still fluctuate about zero.

$$\delta N_X \equiv N_X - \overline{N_X}$$

- So we define susceptibilities

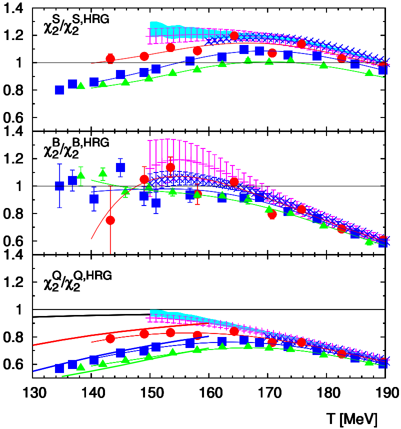
$$\chi_2^X = \langle(\delta N_X)^2\rangle / VT^3$$

- For $X =$ baryon number B, strangeness S, and electric charge Q.
- They are derived from the second-order Taylor coefficients in the expansion of the pressure in terms of chemical potentials.

$$\chi_2^X \equiv \frac{\chi_2^X}{T^2} = \left.\frac{\partial^2 p/T^4}{\partial\hat{\mu}_X^2}\right|_{\vec{\mu}=0} ,$$

$$\chi_{11}^{XY} \equiv \frac{\chi_{11}^{XY}}{T^2} = \left.\frac{\partial^2 p/T^4}{\partial\hat{\mu}_X\partial\hat{\mu}_Y}\right|_{\vec{\mu}=0} ,$$

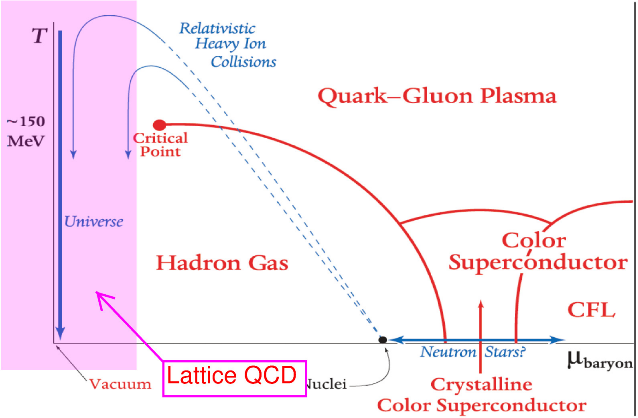
Fluctuations



[HotQCD arXiv:1203.0784]

- Compares the HRG with QCD. The magenta bars and cyan bands show results of two extrapolations to zero lattice spacing.
- We see that the HRG agrees reasonably well for B and Q, but not S.

Reach of Lattice QCD



What Lattice QCD has taught us

- We have learned a lot qualitatively about QCD in thermal equilibrium at low chemical potential for a few flavors and nonzero quark masses.
- We have fairly good control of a variety of some important quantities needed for hydrodynamic modeling.
- We have good quantitative predictions for fluctuations in conserved charges.

What Lattice QCD may still teach us

- We need better ideas/methods for dealing with higher baryon density.
- We hope to learn more about whether the critical endpoint is accessible to experiment.
- We expect to learn more about transport properties: viscosity, electric conductivity, etc. This is difficult, though.
- We don't yet have a completely satisfactory understanding of what happens at the chiral critical point at low $m_u = m_d$, but this will come.
- We expect to learn more about the behavior of the QGP in strong magnetic fields.