# High Temperature and Density in Lattice QCD: Deconfining transition


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# Free energy of a static charge

- Polyakov loop operator

$$L(\mathbf{x}) = P \exp\left[\int igA_0(\mathbf{x}, \tau) d\tau\right]$$

- Expectation value

$$\langle L(\mathbf{x})\rangle = \int [dU] L(\mathbf{x}) \exp[-S_{\rm eff}(U)] / \int [dU] \exp[-S_{\rm eff}(U)].$$

- This operator builds a static external point source, so the expectation value gives the difference in free energy between the ensemble plus an additional static charge and the unmodified ensemble.

$$\exp(-F_0/T) = \langle L \rangle$$

- Actually $F_0 = F_0(a, T)$ depends on the lattice spacing and temperature.
- $F_0(a, T)$ is ultraviolet divergent ($\sim \mathrm{const}/a$), just as in QED.
- Usually, we renormalize so $F_0(T) \equiv F_0(a, T) - F_0(a, T_0) + \mathrm{const}$

## Exercise

*The Wilson fermion action for a fermion of bare mass m is*

$$
\begin{aligned}
S_F &= \sum_{x,x'} \bar{\psi}(x) M(x, x' \psi(x') = \sum_{x} \bar{\psi}(x)\psi(x) \\
&- \kappa \sum_{x,\mu} [\bar{\psi}(x)(1+\gamma_\mu) U_\mu(x)\psi(x+\hat{\mu}) + \bar{\psi}(x+\hat{\mu})(1-\gamma_\mu) U^\dagger_\mu(x)\psi(x)].
\end{aligned}
$$

*where* $\kappa = 1/(8 + 2ma)$. *The fermion propagator is* $M^{-1}(x, x')$. *Note that* $M = 1 - \kappa H$, *where H is called the "hopping matrix". For large bare mass (small* $\kappa$*)* $[1 - \kappa H]^{-1}$ *can be evaluated as a geometric series (hopping parameter expansion). Find the propagator in leading order in* $\kappa$ *for a static quark over the time interval* $[0, t]$.

*The partition function in the presence of a static quark at* $\mathbf{x}$ *is* $\int [dU] \exp(-S) \,\mathrm{Tr}\, M^{-1}(\mathbf{x}, 1/T; \mathbf{x}, 0)$ *where the trace of the propagator is over color and spin.*

*So show that* $\exp(-F_0/T)$ *is proportional to the Polyakov loop operator, where* $F_0$ *is the free energy of a static quark, i.e.,the difference in the free energies of the ensembles with the static quark and without.*

# Free energy of a pair of static charges

- Found through

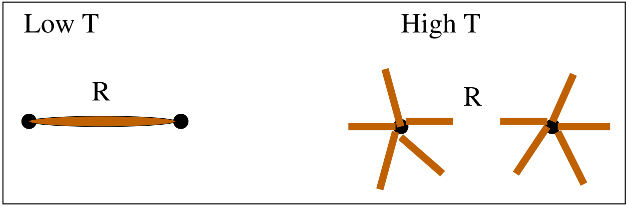
$$\exp[-F(\mathbf{R}, T, a)/T] = \left\langle L(\mathbf{x})L^\dagger(\mathbf{x}+\mathbf{R})\right\rangle$$

- At zero $T$ this is the same as the static quark potential $V(R)$.
- Confinement $R \to \infty$: $F(R) \to \sigma R$ (area law with area $R/T$).
- Dynamical quarks screen the charges, so we always have, asymptotically,

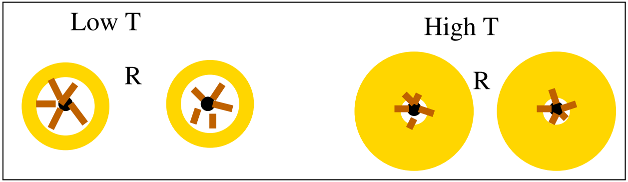
$$F(R, T, a) \to 2F_0(a, T)$$

# Static quark free energy

- Free energy of a static quark and antiquark pair $V(T, R)$
- Consider $R \to \infty$: $V(T, R) \to 2F_q(T)$
- **Pure glue:** $F_q(T)$ is infinite at low $T$ — confined
- Finite at high $T$ — deconfined. First-order phase transition.

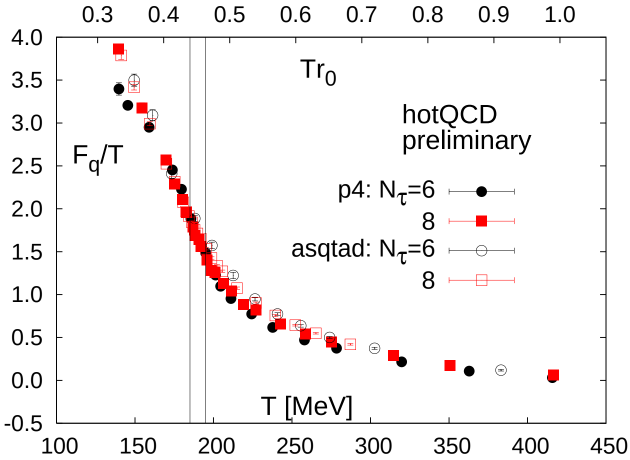



- **With sea quarks:** $F_q(T)$ is finite at all $T$.
- If quarks are light enough, only crossover.

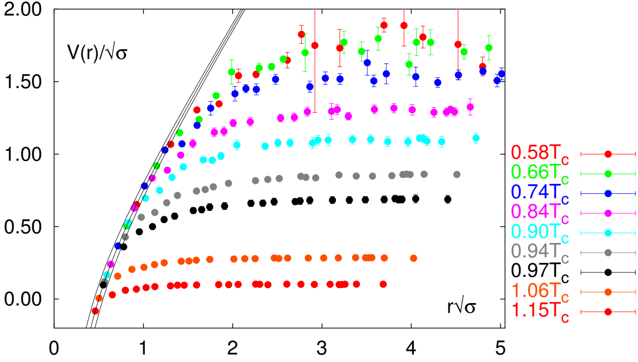


# Static quark free energy

- In the case illustrated below, the quarks are not infinitely massive
- $F_q(T)$ still decreases rapidly with increasing $T$
- Only a qualitative indicator of deconfinement



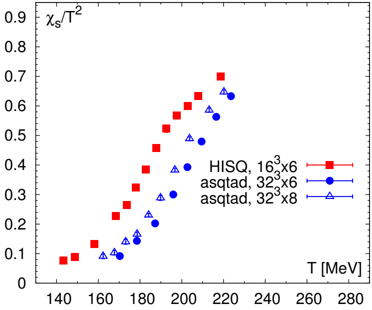
# Free energy of a charge pair




- Karsch, Laermann, Peikert, Nucl.Phys. **B605** (2001) 579.
- $N_f = 3$ with fixed $m_q = 0.1$. $\sigma$ is the string tension.
- Band of lines: Cornell phenomenological heavy quark potential

# Strange quark number susceptibility

- Also a qualitative indicator of deconfinement.
- $\chi_s = \langle N_s^2 \rangle / (VT)$ measures fluctuations in strangeness $N_s$.
- At high $T$ the strange degrees of freedom are light – more fluctuations.




[HotQCD 2010]

# Dimensional Reduction

- Euclidean time boundary conditions

$$\begin{aligned} A_\mu^a(\mathbf{x},\tau) &= A_\mu^a(\mathbf{x},\tau+1/T) \quad \text{periodic} \\ q(\mathbf{x},\tau) &= -q(\mathbf{x},\tau+1/T) \quad \text{antiperiodic} \end{aligned}$$

- Fourier decomposition in imaginary time $\tau$ is

$$\begin{aligned} A_\mu^a(\mathbf{x},\tau) &= \sum_{n=-\infty}^{\infty} \exp(i\omega_{b,n}\tau) A_{\mu,n}^a(\mathbf{x}) \quad \text{for } \omega_{b,n} = 2\pi n T \\ q(\mathbf{x},0) &= \sum_{n=-\infty}^{\infty} \exp(i\omega_{f,n}\tau) q_n(\mathbf{x}) \quad \text{for } \omega_{f,n} = 2\pi(n+\tfrac{1}{2})\, T \end{aligned}$$

- Free-field Euclidean mass-shell condition

$$p_x^2 + p_y^2 + p_z^2 + \omega_n^2 + m^2 = 0.$$

# Turning the Euclidean lattice on its side

In a Euclidean world, any direction can be called imaginary time. So swap $z$ and $\tau$ and let $E = ip_z$.

- Free-field mass-shell condition

$$E^2 = p_x^2 + p_y^2 + \omega_n^2 + m^2.$$

- Tower of 3D bosonic fields

$$E_n^2 = p_x^2 + p_y^2 + m_b^2 + (2\pi n T)^2$$

- Tower of 3D fermionic fields

$$E_n^2 = p_x^2 + p_y^2 + m_f^2 + [2\pi(n + \tfrac{1}{2})T]^2$$

- Three-dimensional Euclidean field theory
  - $A^a_{n,0}$ become scalar fields.
  - $A^a_{n,i}$ are 3D vector fields.
  - $q_n$ are effectively massive fermion fields.
- At high $T$ all fermion fields have high mass regardless of $m_f$.
- Only the $n = 0$ bosons are massless when $m_b = 0$.

# Consequences of dimensional reduction

- Confining zero-temperature 3D Euclidean Gauge-Higgs field theory!
- 3D coupling $g\sqrt{T}$.
- Area law for Wilson loop. Corresponds to space-like Wilson loop in 4D.
- Confinement effects for momenta less than $g^2 T$.
- Confined states in 3D correspond to spatial screening in 4D.

$$\langle A(0)B(\mathbf{r})\rangle \to \exp(-\mu r)/r$$

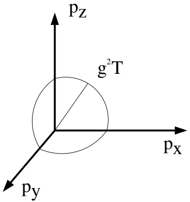
- At high $T$ for $A = \bar{q}\Gamma q$ we have $\mu \approx 2\pi T$.
- QCD exhibits spatial confinement even at the highest $T$!

# Consequences of dimensional reduction

- Thermodynamic potential in perturbation theory

$$\Omega(T) = c_0(T) + \alpha_s c_1(T) + \alpha_s^{3/2} c_{3/2}(T) + \alpha_s^2 c_2(T) + \dots .$$

- Nonperturbative contributions start at order $\alpha_s^3$.
- Volume of momentum space with $p < g^2 T$ goes like $g^6 T^3$.



# Can hadrons survive in the quark plasma?

- Static quark pair might still be bound: charmonium?
- Could residual confinement effects stabilize resonances?
- Wouldn't high temperatures destroy all resonances?

# Spectral functions

- Consider the thermal correlator

$$\left\langle \mathcal{O}^\dagger(\mathbf{x},0)\mathcal{O}(\mathbf{y},\tau)\right\rangle$$

- Do spatial Fourier transform (conserved momentum)

$$C(p,\tau,T) = \left\langle \mathcal{O}^\dagger(\mathbf{p},0)\mathcal{O}(\mathbf{p},\tau)\right\rangle$$

- Spectral decomposition

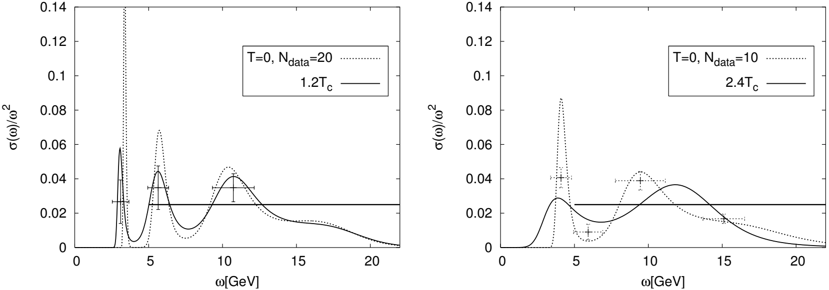
$$C(p,\tau) = \frac{1}{2\pi}\int_0^\infty d\omega\,\rho(\omega,p,T)K(\omega,\tau,T)$$

- Kernel function

$$K(\omega,\tau,T) = \frac{\cosh\omega(\tau - 1/2T)}{\sinh(\omega/2T)}.$$

- The spectral density $\rho(\omega,p,T)$ has peaks in $\omega$ at resonances.

# Spectral functions




[Jakovac, Petreczky, Petrov, Velytsky, Phys.Rev. D75 (2007) 014506]

- Perhaps charmonium survives at $1.2\,T_c$ but not at $2.4\,T_c$?.

# Numerical challenge

- Recall
$$C(p,\tau) = \frac{1}{2\pi}\int_0^\infty d\omega\,\rho(\omega,p,T)K(\omega,\tau,T)$$
- Note that $C(p,\tau,T)$ is measured only for discrete $\tau = 0,1,\ldots,N_\tau - 1$
- But $\rho(\omega,p,T)$ has values for a continuous $\omega$.
- Ill-posed problem.
- Need high precision and MANY imaginary time points. (Lattice with $a_t \ll a_s$ also good!)
- Add extra constraints. A popular one goes by the name maximum entropy.
- Used also to extract transport coefficients: electrical conductivity, shear and bulk viscosity, important for hydrodynamics.